Finding Derivatives Algebraically

Exercise 11, Page 76

Find $f'(-2)$ if $f(x) = x^3$ (find the derivative of a function at a particular point)

$$f'(-2) = \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h}$$

$$= \lim_{h\to 0} \frac{(-2+h)^3 - (-2)^3}{h}$$

$$= \lim_{h\to 0} \frac{[(-2)^3 + 3(-2)^2 h + 3(-2)h^2 + h^3] - (-2)^3}{h}$$

$$= \lim_{h\to 0} \frac{-8 + 12h - 6h^2 + h^3 + 8}{h}$$

$$= \lim_{h\to 0} \frac{12h - 6h^2 + h^3}{h} = \lim_{h\to 0} \frac{h(12 - 6h + h^2)}{h}$$

$$= \lim_{h\to 0} (12 - 6h - h^2) = 12$$

$$\therefore f'(-2) = 12$$

Now find $f'(x)$, the general formula for the first derivative of $f(x) = x^3$, then use this result to find $f'(-2)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - (x)^3}{h}$$

$$= \lim_{h\to 0} \frac{[x^3 + 3(x)^2 h + 3(x)h^2 + h^3] - (x)^3}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + 3x^2 h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{3x^2 h + 3xh^2 + h^3}{h} = \lim_{h\to 0} \frac{h(3x^2 + 3xh + h^2)}{h}$$

$$= \lim_{h\to 0} (3x^2 + 3xh + x^2)$$

$$f'(x) = 3x^2$$

$$f'(-2) = 3(-2)^2 = 12$$