The Product Rule

If $y = f(x)$ and $y = g(x)$ are differentiable, then $(f \cdot g)' = f' \cdot g + f \cdot g'$.

That is, *the derivative of a product equals the derivative of the first function times the second function, plus the first function times the derivative of the second function.*

Alternative notation: $\frac{d}{dx}[f \cdot g] = \frac{df}{dx} \cdot g + f \cdot \frac{dg}{dx}$.

Exercise 5, Page 121.

Find f' if $f(x) = \sqrt{x} \cdot 2^x$.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[\sqrt{x} \cdot 2^x\right] \\ &= \frac{d}{dx}\left[\sqrt{x}\right] \cdot 2^x + \sqrt{x} \cdot \frac{d}{dx}\left[2^x\right] \\ &= \left[\frac{1}{2} x^{-1/2}\right] \cdot 2^x + \sqrt{x}\left[(\ln 2) \cdot 2^x\right] \\ &= \frac{2^x}{2\sqrt{x}} + \sqrt{x}\left[(\ln 2) \cdot 2^x\right] \end{aligned}$$

Note: $\sqrt{x} = x^{1/2} \Rightarrow \frac{d}{dx}\left[\sqrt{x}\right] = \frac{d}{dx}\left[x^{1/2}\right] = \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

Exercise 10, Page 121

Find f' if $f(x) = \frac{x}{e^x}$.

Change $f(x) = \frac{x}{e^x}$ to a product and use the product rule: $f(x) = \frac{x}{e^x} = x \cdot e^{-x}$

$$\begin{aligned} f'(x) &= \frac{d}{dx}[x] \cdot e^{-x} + x \cdot \frac{d}{dx}\left[e^{-x}\right] \\ &= [1] \cdot e^{-x} + x \cdot e^{-x}(-1) \\ &= e^{-x} - xe^{-x} \\ &= e^{-x}(1 - x) \\ &= \frac{1 - x}{e^x} \end{aligned}$$