The Quotient Rule

If $y = f(x)$ and $y = g(x)$ are differentiable, then

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{\frac{df}{dx}\cdot g - f\cdot\frac{dg}{dx}}{[g]^2}$$

Alternative notation: $$\left[\frac{f}{g}\right]' = \frac{f'\cdot g - f\cdot g'}{[g]^2}$$

Be careful! Note that the numerator is a difference and differences, unlike sums, are NOT COMMUTATIVE.

Tip: It might be easier if you always do the product rule *and* the numerator of the quotient in the quotient rule in the same order (that is, the derivative of the first times the second, then the first times the derivative of the second), and also remember that the product rule is a SUM and the quotient rule's numerator is a DIFFERENCE.

Exercise 10, Page 121

$$\frac{d}{dx}\left[\frac{x}{e^x}\right] = \frac{\frac{d[x]}{dx}\cdot e^x - x\cdot\frac{d[e^x]}{dx}}{\left[e^x\right]^2}$$

$$= \frac{1\cdot e^x - x\cdot e^x}{e^{2x}}$$

$$= \frac{e^x(1-x)}{e^{2x}}$$

$$= \frac{1-x}{e^x}$$

See **The Product Rule – Page 1** for an alternative method.

Exercise 25, Page 121

$$\frac{d}{dx}\left[\frac{17e^x}{2^x}\right] = \frac{(17e^x)'\cdot 2^x - 17e^x\cdot(2^x)'}{\left(2^x\right)^2}$$

$$= \frac{(17e^x)\cdot 2^x - 17e^x(\ln 2)\cdot 2^x}{\left(2^x\right)^2}$$

$$= \frac{17e^x 2^x\left[1-\ln 2\right]}{\left(2^x\right)^2}$$

$$= \frac{17e^x\left[1-\ln 2\right]}{2^x}$$