TECHNIQUES OF INTEGRATION

INTEGRATION BY SUBSTITUTION: Used to "undo" the results of the <u>chain rule</u>.

Chain rule: $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$

To integrate $\int f'(g(x)) \cdot g'(x)\,dx$ use substitution.

Let $u = g(x)$, the "inside" function.

Then $\frac{du}{dx} = g'(x) \;\Rightarrow\; du = g'(x)dx \;\Rightarrow\; dx = \frac{du}{g'(x)}$.

Now replace $g(x)$ with u and dx with $\frac{du}{g'(x)}$.

$$\begin{aligned}\int f'(g(x)) \cdot g'(x)\,dx &= \int f'(u) \cdot g'(x) \cdot \frac{du}{g'(x)} \\ &= \int f'(u)\,du \\ &= f(u) + C \\ &= f(g(x)) + C\end{aligned}$$

Note: The entire integral must be in terms of u; no x's should remain. If this is not the case, try a different choice for u.

INTEGRATION BY PARTS: Used to "undo" the results of the <u>product rule</u>.

Product Rule: $\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

To generate the formula for integration by parts, integrate both sides and solve for the integral that has $f(x) \cdot g'(x)$ as its integrand.

$$\int \frac{d}{dx}\left[f(x) \cdot g(x)\right]dx = \int f'(x) \cdot g(x)\,dx + \int f(x) \cdot g'(x)\,dx$$

$$f(x) \cdot g(x) = \int f'(x) \cdot g(x)\,dx + \int f(x) \cdot g'(x)\,dx$$

$$\int f(x) \cdot g'(x)\,dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x)dx$$

Rewrite the above formula by choosing

$u = f(x)$ so that $u' = f'(x)$

$v' = g'(x)$ so that $v = g(x)$

$$\int uv'\,dx = u\,v - \int u'v\,dx$$

The key to integrating by parts is that whatever you choose for v', you must be able to find its antiderivative and this antiderivative should be simpler than v'.